ZPF, Zitterbewegung and Inertial Mass

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Abstract

In this paper, we argue that the background Zero Point Field, induces an oscillatory motion which we identify with zitterbewegung. We deduce that this is the origin of inertial mass.

Keywords: ZPF, Zitterbewegung, Mass.

1 Introduction

We would first like to point out that a background Zero Point Field of the kind we have been considering can explain the Quantum Mechanical spin half (as also the anomalous $g = 2$ factor) for an otherwise purely classical electron [1, 2, 3]. The key point here is (Cf.ref.[1]) that the classical angular momentum $\vec{r} \times m\vec{v}$ does not satisfy the Quantum Mechanical commutation rule for the angular momentum $\vec{J}$. However when we introduce the background Zero Point Field (ZPF), the momentum now becomes

$$\vec{J} = \vec{r} \times m\vec{v} + (e/2c)\vec{r} \times (\vec{B} \times \vec{r}) + (e/c)\vec{r} \times \vec{A}^0, \qquad (1)$$

where $\vec{A}^0$ is the vector potential associated with the ZPF and $\vec{B}$ is an external magnetic field introduced merely for convenience, and which can be made vanishingly small.

It can be shown that $\vec{J}$ in (1) satisfies the Quantum Mechanical commutation relation for $\vec{J} \times \vec{J}$. At the same time we can deduce from (1)

$$\langle J_z \rangle = -\frac{1}{2}\hbar\omega_0/|\omega_0| \qquad (2)$$

Relation (2) gives the correct Quantum Mechanical results referred to above. From (1) we can also deduce that

$$l = \langle r^2 \rangle^{\frac{1}{2}} = \left(\frac{\hbar}{mc} \right) \quad (3)$$

Equation (3) shows that the mean dimension of the region in which the ZPF fluctuation contributes is of the order of the Compton wavelength of the electron. By relativistic covariance (Cf.ref.[2]), the corresponding time scale is at the Compton scale. Dirac, in his relativistic theory of the electron encountered the zitterbewegung effects within the Compton scale [4] and he had to invoke averages over this scale to recover meaningful results. We on the other hand have shown elsewhere, without invoking the ZPF [5] how spin follows, that is we get (2) and (3) using zitterbewegung conclusions. We would next like to show that the ZPF leads to the mass, that is, inertial mass of a particle.

2 Modelling the ZPF

We can model the ZPF in terms of a Weiner process [6]. In this case the right hand and left hand time derivatives at any instant are unequal. Let us push these considerations further. We would like to point out that it would be reasonable to expect that the Weiner process is related to the ZPF which is the Zero Point Energy of a Quantum Harmonic oscillator. We can justify this expectation as follows: Let us denote the forward and backward time derivatives with respect to time by d_+ and d_-. In usual theory where time is differentiable, these two are equal, but we have on the contrary taken them to be unequal. Let us consider the simple case,

$$d_- = a - d_+ \quad (4)$$

Then we have from Newton's second law in the absence of forces,

$$\ddot{x} + k^2 x = a\dot{x} \quad (5)$$

wherein the new nondifferentiable effect (4) is brought up. In a normal vacuum with usual derivatives and no external forces, Newtonian Mechanics would give us instead the equation

$$\ddot{x} = 0 \quad (6)$$

A comparison of (5) and (6) shows that the Weiner process converts a uniformly moving particle, or a particle at rest into an oscillator– a damped oscillator, strictly speaking. Indeed in (5) if we take as a first approximation

$$\dot{x} \approx \langle \dot{x} \rangle = 0 \tag{7}$$

then we would get the exact oscillator equation

$$\ddot{x} + k^2 x = 0 \tag{8}$$

for which in any case, consistently (7) is correct. Moreover, the ZPF results from the quantized oscillator (8). We will return to this a little later.
We can push these considerations even further and deduce alternatively, the Schrodinger equation, as pointed out by Nottale [7]. The genesis of Special Relativity too can be found in the Weiner process (Cf.ref.[5] for a detailed discussion). Let us examine this more closely.
We first define a complete set of base states by the subscript $\imath$ and $U(t_2, t_1)$ the time elapse operator that denotes the passage of time between instants t_1 and t_2, t_2 greater than t_1. We denote by, $C_\imath(t) \equiv < \imath|\psi(t) >$, the amplitude for the state $|\psi(t) >$ to be in the state $|\imath >$ at time t, and [8, 9]

$$< \imath|U|j > \equiv U_{\imath j}, U_{\imath j}(t + \Delta t, t) \equiv \delta_{\imath j} - \frac{\imath}{\hbar} H_{\imath j}(t)\Delta t.$$

We can now deduce from the super position of states principle that,

$$C_\imath(t + \Delta t) = \sum_j [\delta_{\imath j} - \frac{\imath}{\hbar} H_{\imath j}(t)\Delta t] C_j(t) \tag{9}$$

and finally, in the limit,

$$\imath\hbar \frac{dC_\imath(t)}{dt} = \sum_j H_{\imath j}(t) C_j(t) \tag{10}$$

where the matrix $H_{\imath j}(t)$ is identified with the Hamiltonian operator. It must be mentioned that $H_{\imath j}$ gives the probability for a state $C_\imath$ to be found in state C_j. Such a non-local transition is allowed within the Compton scale, as discussed in detail by Weinberg [10]. As Weinberg notes:"Although the relativity of temporal order raises no problems for classical physics, it plays a profound role in quantum theories. The uncertainty principle tells us that when we specify that a particle is at position x_1 at time t_1, we cannot also

define its velocity precisely. In consequence there is a certain chance of a particle getting from x_1 to x_2 even if $x_1 - x_2$ is space-like, that is, $|x_1 - x_2| > |x_1^0 - x_2^0|$. To be more precise, the probability of a particle reaching x_2 if it starts at x_1 is nonnegligible as long as

$$0 \leq (x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 \leq \frac{\hbar^2}{m^2} \cdots \text{"}$$

We have argued earlier at length that (10) leads to the Schrodinger equation [8, 9]. In this derivation, the mass term in the Schrodinger equation comes from $H_{\imath j}$: The matrix $H(x, x')$ gives the probability amplitude for the particle at x to be found at x', that is,

$$H(x, x') =< \psi(x')|\psi(x) > \qquad (11)$$

where as is usual we write $C(x) \equiv \psi(x)(\equiv |\psi(x) >)$, the state of a particle at the point x).
Usually the amplitude $H(x, x')$ is non-zero only for neighbouring points x and x', that is, $H(x, x') = f(x)\delta(x - x')$. But if $H(x, x')$ is not of this form, then there is a non-zero amplitude for the particle to "jump" to an other than neighbouring point. In this case $H(x, x')$ may be described as a non local amplitude. However, in the light of the above, we consider such a non local behaviour, only within the Compton scale.
The contribution of this term is,

$$\int \psi^*(x')\psi(x)\psi(x')U(x')dx'$$

where,
i)$U(x) = 1$ for $|x| < R, R$ arbitrarily large and also $U(x)$ falls off rapidly as $|x| \to \infty; U(x)$ has been introduced merely to ensure the convergence of the integral; and
ii)$H(x, x') =< \psi(x')\psi(x) >= \psi^*(x')\psi(x)$.
The presence of the, what at first sight may seem troublesome, non-linear and non-local term above.
Finally we get

$$m_0 = \int \psi^*(x')\psi(x')U(x')dx' \qquad (12)$$

(Cf.[9] for details).
In the above we have taken the usual unidirectional time to deduce the non

relativistic Schrodinger equation. If however we consider a Weiner process in (9) then we will have to consider instead of (10)

$$C_\imath(t-\Delta t) - C_\imath(t+\Delta t) = \sum_j \left[\delta_{\imath j} - \frac{\imath}{\hbar} H_{\imath j}(t)\Delta t\right] C_j(t) \qquad (13)$$

Equation (13) in the limit can be seen to lead to the relativistic Klein-Gordon equation rather than the Schrodinger equation with the mass term [11]. Furthermore, the Klein-Gordon equation describes the normal mode vibrations of Harmonic Oscillators–that is, we recover (8), again. Thus the ZPF modelled by a Weiner process leads to the mass term at the Compton scale, and indeed, special relativity. In other words, the mass arises due to the "bootstrapping" effect within the Compton scale that arises due to the ZPF or zitterbewegung.

3 An Alternative Derivation

To look at the above from a different point of view, let us start with the Langevin equation in the absence of external forces,[12, 13]

$$m\frac{dv}{dt} = -\alpha v + F'(t)$$

where the coefficient of the frictional force is given by Stokes's Law [14]

$$\alpha = 6\pi\eta a$$

η being the coefficient of viscosity, and where we are considering a sphere of radius a.
For t, we consider in the above spirit of equation (3), that there is a cut off time τ. It is known (Cf.[12]) that there is a characteristic time constant of the system, given by

$$\frac{m}{\alpha} \sim \frac{m}{\eta a},$$

so that, from Stokes's Law, as

$$\eta = \frac{mc}{a^2} \, \text{or} \, m = \eta\frac{a^2}{c}$$

we get

$$\tau \sim \frac{ma^2}{mca} = \frac{a}{c},$$

that is τ is the Compton time.
The expression for η which follows from the fact that

$$F_x = \eta(\Delta s)\frac{dv}{dz} = m\dot{v} = \eta\frac{a^2}{c}\dot{v},$$

shows that the intertial mass is due to a type of "viscosity" of the background Zero Point Field (ZPF). (Cf. also ref.[15]).
To push these small scale considerations further, we have, using the Beckenstein radiation equation[16],

$$t \equiv \tau = \frac{G^2m^3}{\hbar c^4} = \frac{m}{\eta a} = \frac{a}{c}$$

which gives

$$a = \frac{\hbar}{mc} \quad \text{if} \quad \frac{Gm}{c^2} = a$$

In other words the Compton wavelength equals the Schwarzchild radius, which automatically gives us the Planck mass. Thus the inertial mass is thrown up in these considerations at the Planck scale.
In fact if we now use the Langevin equation in a viscous medium [17, 13] then as the viscosity becomes vanishingly small, it turns out that the Brownian particle moves according to Newton's first law, that is with a constant velocity. Moreover this constant velocity is given by (Cf.refs. [17, 13]), for any mass m,

$$\langle v^2 \rangle = \frac{kT}{m} \tag{14}$$

We can derive that this velocity is that of light in a heuristic way. We have already seen that the ZPF causes a harmonic motion. Let us assume that the particle has a small charge e, just to couple it to the ZPF. The equation of motion is now given by (Cf.[18])

$$\ddot{x} + \omega^2 J = (e/m)E_x^0$$

along the x axis, where, suppressing the polarization states for the moment, the random field $\vec{E}$ is given by

$$\vec{E} \propto \int d^3\vec{K} exp[-\imath(\omega t - K \cdot \vec{r})]\vec{a}_K + c \cdot c \cdot$$

where, owing to the randomness in phase, their averages vanish. What this means is that, finally,

$$L^2 = \langle x^2 \rangle = \hbar/(m\omega) \, \text{and} \, \langle \dot{x^2} \rangle = (\hbar/m)\,\omega = v^2 \tag{15}$$

In (15) above, the frequency is given by,

$$\omega = mc^2/\hbar$$

Whence, within the Compton length,

$$\langle \dot{x}^2 \rangle = c^2$$

Using this result in (14), we get for the Planck mass, the correct radiation for the Beckenstein temperature. For an elementary particle in general, T would be the well known Hagedorn temperature [19].
We finally make the following remark. We have seen that a particle in the ZPF modeled by a Random electromagnetic field, leads to equation (15), viz.,

$$L^2 = \langle x^2 \rangle = \hbar/m\omega,$$
$$\langle \dot{x^2} \rangle = (\hbar/m)\omega,$$

where the frequency is given $\omega = mc^2/\hbar$. In the equation leading to (15), we could also include a term which gives the third derivative of x, this corresponding to the Schott term of classical electrodynamics [20]. However the contribution of this term, which was introduced for energy conservation to compensate the radiation loss of an accelerated charge [20] in the classical electron theory, is of the order of the Compton wavelength [21] and does not effect the conclusion.
The above show that there is oscillation of the particle within the Compton wavelength L, with velocity c. This as mentioned, models the well known zitterbewegung of Dirac where also, the electron has the speed of light within the Compton wavelength. However, what all this means is that via the Compton length L, we get the inertial mass of the particle, which is now seen to be due to the energy of this oscillation.
A final comment. Following Wheeler, we can consider the ZPF in terms of the fluctuations of the vacuum electromagnetic field. If this field vanishes everywhere except in a region of dimension l, then the energy is given by

$$\text{Energy} \sim \hbar c/l$$

If l is the Compton wavelength, this turns out to be mc^2 [6]. This reconfirms our above conclusion.

References

[1] Sachidanandam, S. (1983). *Physics Letters* Vol.97A, No.8, 19 September 1983, pp.323–324.

[2] Sidharth, B.G. (2005). *The Universe of Fluctuations* (Springer, Netherlands).

[3] Milonni, P.W. (1994). *The Quantum Vacuum: An Introduction to Quantum Electrodynamics* (Academic Press, San Diego).

[4] Dirac, P.A.M. (1958). *The Principles of Quantum Mechanics* (Clarendon Press, Oxford), pp.4ff, pp.253ff.

[5] Sidharth, B.G. (2002). *Found. of Phys.Lett.* 15 (5), 501ff.

[6] Sidharth, B.G. (2008). *The Thermodynamic Universe* (World Scientific), Singapore.

[7] Nottale, L. (1993). *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity* (World Scientific, Singapore), pp.312.

[8] Sidharth, B.G. (1997). *Ind.J. of Pure and Applied Physics* 35, pp.456ff.

[9] Sidharth, B.G. (2001). *Chaotic Universe: From the Planck to the Hubble Scale* (Nova Science, New York).

[10] Weinberg, S. (1972). *Gravitation and Cosmology* (John Wiley & Sons, New York), p.61ff.

[11] Sidharth, B.G. (2002). *Chaos, Solitons and Fractals* 13, pp.189–193.

[12] Reif, F. (1965). *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill Book Co., Singapore).

[13] Balescu, R. (1975). *Equilibrium and Non Equilibrium Statistical Mechanics* (John Wiley, New York).

[14] Joos, G. (1951). *Theoretical Physics* (Blackie, London), pp199ff.

[15] Rueda, A. and Haisch, B. (1998). *Found. of Phys.* Vol.28, No.7, pp.1057–1108.

[16] Ruffini, R. and Zang, L.Z. (1983). *Basic Concepts in Relativistic Astrophysics* (World Scientific, Singapore), p.111ff.

[17] Sidharth, B.G. (2006). *Found.Phys.Lett.* **19**, 1, pp.87ff.

[18] Sachidanandam, S. and Raghavacharyulu, I.V.V. (1983). *Ind.J.Pure and Appl.Phys.*, Vol.21, July 1983, pp.408–412.

[19] Sivaram, C. (1982). *Astrophysics and Space Science* 88, pp.507–510.

[20] Rohrlich, F. (1965). *Classical Charged Particles* (Addison-Wesley, Reading, Mass.), pp.145ff.

[21] Sidharth, B.G. (1999). *Instantaneous Action at a Distance in Modern Physics: "Pro and Contra"* A.E. Chubykalo et al. (eds.) (Nova Science Publishing, New York).